Quadrature AM and Quaternary PSK

Consider a general angle modulated signal

$$g(t) = A\cos(\omega_c t + D_p m(t))$$

of the baseband signal $m(t)$. If $m(t)$ is a binary signal then the result is a phase-shift keyed signal $s(t)$.

However, suppose that more than two modulation levels are allowed at the transmitter input. In this case *different* signals are transmitted for different input signal levels. For example, if $M = 4$ levels are allowed at the input, corresponding to angles of 0, 90, 180, and 270 degrees, the possible waveforms transmitted during a symbol interval T_s are

$$s_1(t) = A\cos\omega_c t$$
$$s_2(t) = A\sin\omega_c t$$
$$s_3(t) = -A\cos\omega_c t$$
$$s_4(t) = -A\sin\omega_c t.$$

This is called M-ary phase-shift keying (MPSK) signalling, where M is the number of different signal phases that may be used during a transmission interval.

The scheme outlined above can be described by the complex envelope of the signal

$$g(t) = Ae^{j\theta(t)},$$

which will take on 4 different values corresponding to the four phases that θ is permitted to have. A plot of the permitted values of the complex envelope will therefore contain four points, with each envelope being represented by one complex number in general.

For example, suppose the permitted multilevel values are -3, -1, 1, and 3. The corresponding PSK phases may then be 0, 90, 180, and 270 degrees

respectively, as indicated below:

Imaginary
(quadrature)
$g(t)$

θ

Real
(in phase)

PSfrag replacements
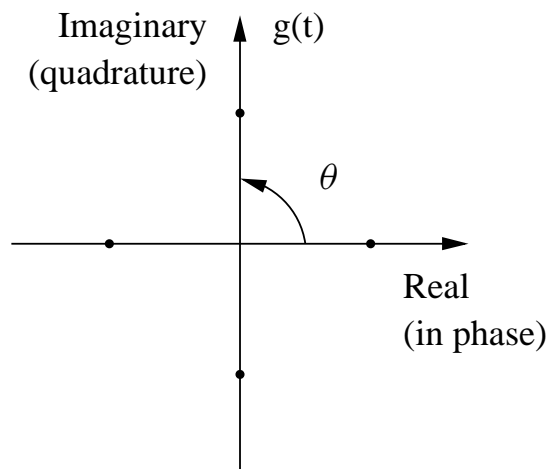
Alternatively, the levels may correspond to angles of 45, 135, 225, and 315 degrees respectively, as indicated by

Imaginary
(quadrature)
$g(t)$

y_i

x_i

Real
(in phase)

PSfrag replacements
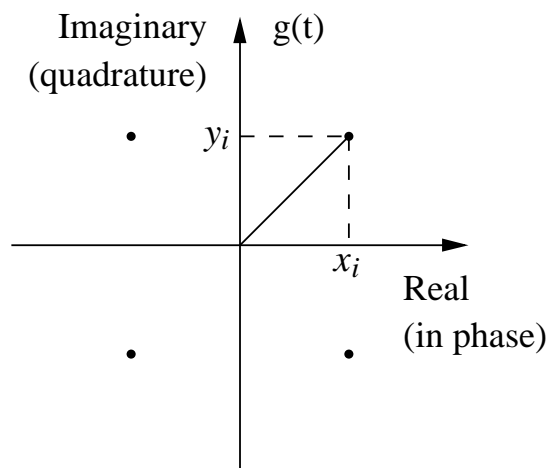
This case differs from the previous only in a shift in the carrier phase reference. This example of M-ary PSK where $M = 4$ is called *quadrature phase-shift-keyed* (QPSK) signalling. The diagrams above are called the QPSK *constellation* diagrams, and contain information regarding the permitted values of the complex envelope.

To calculate the probability of error, we need to propose a form for the demodulator. Because the signal can be considered as two BPSK signals in quadrature, a detector involves two BPSK receivers in parallel, one for each quadrature carrier. Such a system is shown below:
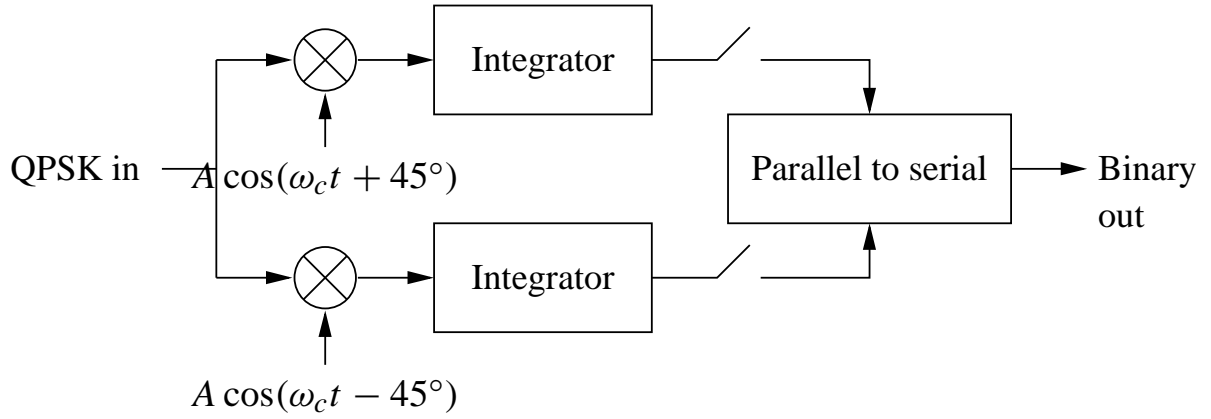
The possible outputs of the first correlator in the absence of noise are

$$y_1(T_s) = \int_0^{T_s} [\pm A \cos \omega_c t][A \cos(\omega_c t + \pi/4] dt$$

$$= \frac{\pm A^2 T_s}{2\sqrt{2}} = \frac{\pm E_s}{\sqrt{2}},$$

with E_s the energy per symbol. Similarly the output from the second correlator is

$$y_2(T_s) = \frac{\pm E_s}{\sqrt{2}}.$$

The probability of the first correlator output being in error can be shown to be

$$P_{\epsilon_1} = \text{erfc} \sqrt{\frac{E_s}{\eta}}$$

and similarly for the second, so $P_{\epsilon_2} = P_{\epsilon_1}$. The probability of correctly identifying the transmitted signal is therefore

$$P_c = (1 - P_{\epsilon_1})(1 - P_{\epsilon_2}),$$

assuming that the noise in the two correlators is independent. Therefore

$$P_c = 1 - 2P_{\epsilon_1} + P_{\epsilon_1}^2 \approx 1 - 2P_{\epsilon_1},$$

and

$$P_\epsilon = 1 - P_c = 2\text{erfc}\sqrt{\frac{E_s}{\eta}}.$$

Since *two* bits are effectively being transmitted per symbol, we have $T_s = 2T_b$, so the effective bit energy is $E_b = E_s/2$. In terms of energy per bit the performance of QPSK is therefore

$$P_\epsilon = \text{erfc}\sqrt{\frac{2E_b}{\eta}},$$

which is the same as for PRK.

The primary advantage of using QPSK is a reduction in the bandwidth used. Since the signal is the result of the linear addition of two DSB-SC signals, the bandwidth used is the same as for each DSB-SC component. Thus the QPSK example outlined above permits the transmission of data at 2 bps/Hz.

MPSK signals can also be generated by using two quadrature carriers modulated by the x and y components of the complex envelope
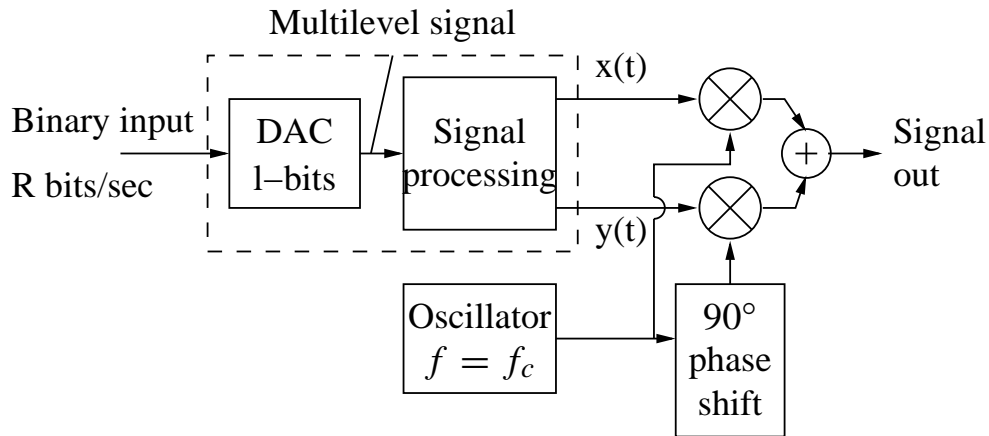
$$g(t) = Ae^{j\theta(t)} = x(t) + jy(t),$$

where the permitted values of x and y are

$$x_i = A\cos\theta_i$$
$$y_i = A\sin\theta_i$$

for the permitted phase angles θ_i, $i = 1, 2, \ldots, M$ of the MPSK signal. This is illustrated in the figure below:

Multilevel signal

				x(t)		

Binary input   DAC   Signal   ⊗   Signal

R bits/sec   l–bits   processing   +   out

                 y(t)   ⊗

Oscillator   90°

$f = f_c$ phase

             shift

PSfrag replacements

As in the case of PRK, QPSK demodulators and detectors require carrier recovery circuits to obtain a reference carrier waveform. A number of methods are available for obtaining this reference:

- The incoming QPSK waveform can be quadrupled and then divided by 4 to obtain the carrier,

- A Costas loop can be used

- A decision-feedback loop can be used,

- The modulated signal can first be demodulated and then remodulated, and phase-compared to a delayed replica of the modulated signal.
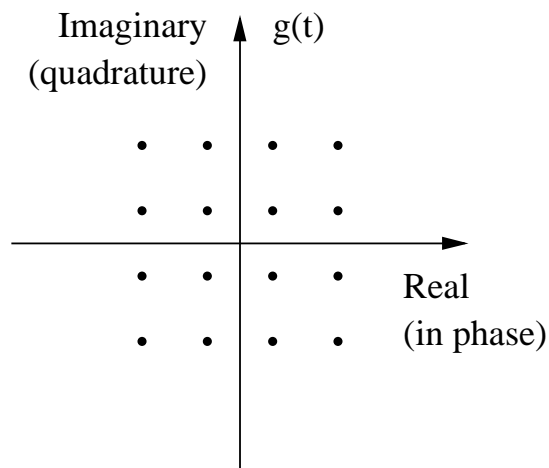
In quadrature amplitude modulation (QAM), the signal constellations are not restricted to having signalling points only on a circle. The general QAM signal is

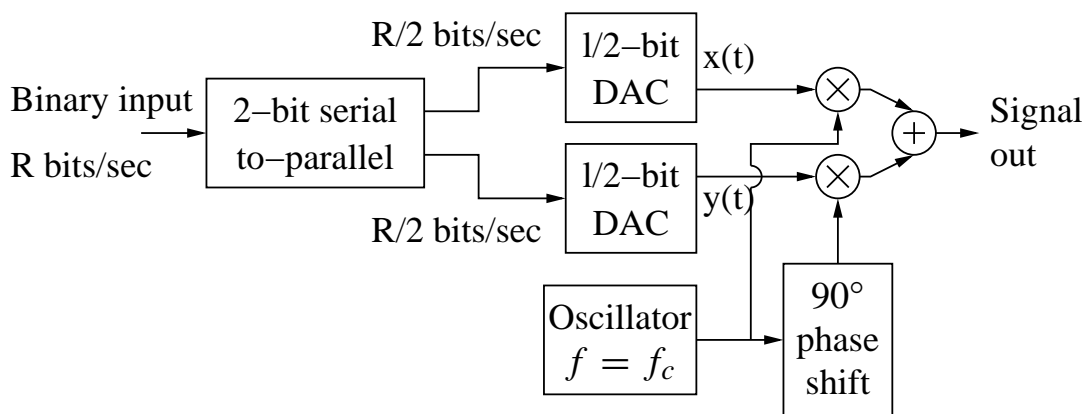$$s(t) = x(t) \cos \omega_c t + y(t) \sin \omega_c t,$$

where

$$g(t) = x(t) + jy(t) = R(t)e^{j\theta(t)}.$$

For example, a popular 16-symbol QAM constellation is

Here x_i and y_i are each permitted to have four levels per dimension. The 16-symbol QAM signal may be generated using a system of the following form:

This scheme is commonly used in 2400 bit/sec V.22 bis computer modems.