

EEE4001F: Digital Signal Processing

Class Test 1

11 March 2010

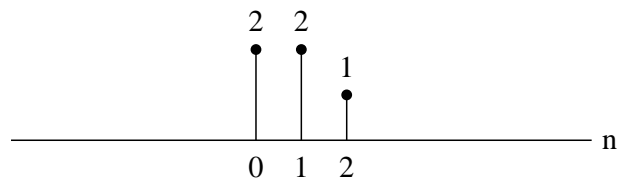
Name:

Student number:

Information

- The test is closed-book.
 - This test has *four* questions, totalling 20 marks.
 - Answer *all* the questions.
 - You have 45 minutes.
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1. (5 marks) If $x[n]$ is the signal



then sketch the following:

(a) $x_1[n] = -x[n + 2]$

(b) $x_2[n] = x[2n + 1]$

(c) $x_3[n] = \begin{cases} x[n/2] & n \text{ even} \\ 0 & n \text{ odd} \end{cases}$

(d) $x_4[n] = \frac{1}{2}(x[n] + x[-n]).$

2. (5 marks) A linear system has the relationship

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]g[n-2k]$$

between its input $x[n]$ and its output $y[n]$, where $g[n] = u[n] - u[n-4]$.

- (a) Determine $y[n]$ when $x[n] = \delta[n-1]$.
- (b) Determine $y[n]$ when $x[n] = \delta[n-2]$.
- (c) Is the system LTI?

3. (5 marks) Consider two systems described by the following linear constant coefficient difference equations:

$$y[n] = 0.2y[n - 1] + x[n] + 0.3x[n - 1] + 0.02x[n - 2]$$

$$y[n] = x[n] - 0.1x[n - 1].$$

Prove that the two systems are equivalent.

4. (5 marks) The input $x[n]$ and the output $y[n]$ of a causal system obeys the relationship

$$y[n] = x[n] + 0.3y[n - 2].$$

Find the impulse response, and determine whether the system is BIBO stable or not.

Fourier transform properties

Sequences $x[n], y[n]$	Transforms $X(e^{j\omega}), Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n - n_d]$	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$	Modulation

Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
1 $(-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n]$ $(a < 1)$	$\frac{1}{1 - ae^{-j\omega}}$
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n + 1)a^n u[n]$ $(a < 1)$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \omega < \omega_c \\ 0 & \omega_c < \omega \leq \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z < 1$
$\delta[n - m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z > a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z < a $
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z > a $
$-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z < a $
$\begin{cases} a^n & 0 \leq n \leq N - 1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1 - a^N z^{-N}}{1 - az^{-1}}$	$ z > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1 - r \cos(\omega_0)z^{-1}}{1 - 2r \cos(\omega_0)z^{-1} + r^2 z^{-2}}$	$ z > r$