EEE4001F EXAM
DIGITAL SIGNAL PROCESSING

University of Cape Town
Department of Electrical Engineering

June 2008
3 hours

Information

- The exam is closed-book.
- There are two parts to this exam.
- **Part A** has *eight* questions totalling 70 marks. You must answer all of them.
- **Part B** has *two* questions, each counting 15 marks. You must answer *both* of them.
- A table of standard z-transform pairs appears at the end of this paper.
- A formula sheet for the radar/sonar question appears at the end of this paper.
- You have 3 hours.

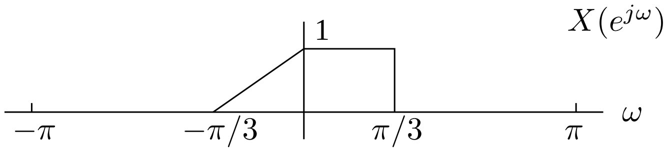
PART A

Answer all of the following questions.

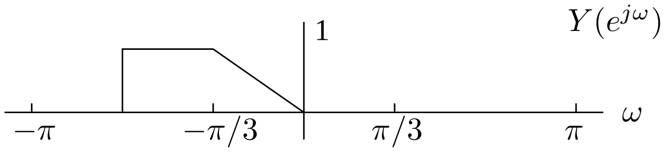
1. If $x[n] = \delta[n-1] + 2\delta[n-2] - \delta[n-4]$, sketch
   (a) $y_1[n] = x[n-2]$,
   (b) $y_2[n] = -2x[n]$,
   (c) $y_3[n] = x[2n]$,
   (d) $y_4[n] = x[n-2] * \delta[n+1]$,
   (e) $y_5[n] = x[n] * x[n]$,
   (f) $y_6[n] = x[n] * x[n-1]$.

(15 marks)

2. A sequence $x[n]$ has a zero-phase DTFT $X(e^{j\omega})$ given below:



The sequence $y[n]$ has the following zero-phase DTFT:



Find an expression for $y[n]$ in terms of $x[n]$.

(8 marks)

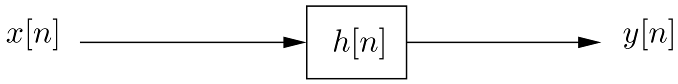
3. Consider the following z-transform:

$$X(z) = \frac{z^{-1} - 2z^{-3} + 2z^{-5} - z^{-7}}{1 - z^{-1}}.$$

Determine the right-sided discrete-time signal $x[n]$ of which $X(z)$ is the z-transform, and sketch your answer for the interval $0 \leq n \leq 8$.

(8 marks)

4. Consider the LTI system with impulse response $h[n]$:



When the input $x[n] = \delta[n] + \delta[n-2]$ is applied to the system, the output is $y[n] = 3\delta[n] + \delta[n-1] + 2\delta[n-2] - \delta[n-3] + \delta[n-4]$.

(a) Determine the z-transform $X(z)$ of $x[n]$.

(b) Determine the z-transform $Y(z)$ of $y[n]$.

(c) Use your previous answers to determine a closed-form expression for the impulse response $h[n]$ for the system under the assumption that it is causal.

Note: you may need the z-transform pair

$$\cos(\omega_0 n)u[n] \overset{z}{\longleftrightarrow} \frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}} \qquad \text{ROC: } |z| > 1.$$

(10 marks)

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5. Find $w[n] = x[n] * y[n]$ with

$$x[n] = \cos(\pi n/2) \quad \text{and} \quad y[n] = \frac{\sin(\pi n/3)}{\pi n}.$$

(5 marks)

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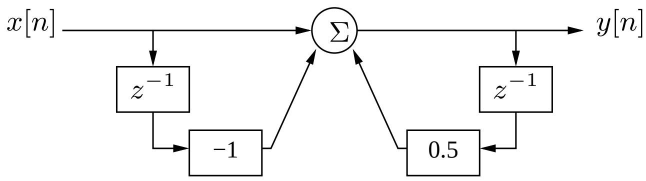
6. Consider the following LTI system:



Determine a closed-form expression for the frequency response $H(e^{j\omega})$ of this system and then use this to sketch the magnitude $|H(e^{j\omega})|$ over the range $0 \leq \omega \leq 2\pi$. Indicate approximate amplitudes at $\omega = 0$, $\omega = \frac{\pi}{4}$, $\omega = \frac{\pi}{2}$, and $\omega = \pi$.

(10 marks)

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7. A signal with a sampling frequency of 8kHz must be resampled to 7.2kHz. Draw a block diagram of a system, using up-samplers, filters, and down-samplers, to achieve this. Indicate clearly the cutoff frequency of any filters. Will any information be lost in this conversion?

(7 marks)

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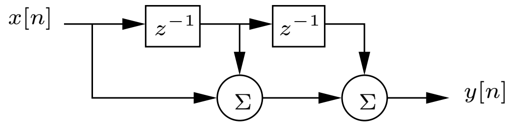
8. Consider the filter structure shown below:



Show that this filter has a linear phase characteristic, and explain why linear phase is important in some signal processing applications.

(7 marks)

PART B

Answer *both* of the following questions. Each question counts 15 marks.

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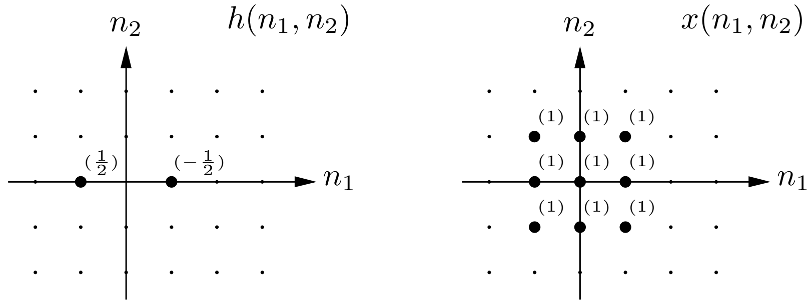
1. **Image processing and computer vision**

   (a) Find and sketch the 2-D convolution between the following two signals:



(5 marks)

   (b) The signal $h(n_1, n_2)$ in the previous question is the impulse response of a 2-D edge detector. Explain why a system with this impulse response is able to detect edges. What edge direction does it detect? Draw the impulse response of a filter that can be used to detect edges in the orthogonal direction. How could you modify these filters to reduce noise in the filter output?

(5 marks)

   (c) In some application you observe 3 data points with the following (x, y) coordinates: $(1, 1)$, $(3, 2)$, $(2, \frac{2}{3})$. You need to fit a straight line through the origin with unknown slope to these points: the parametric family of curves is therefore

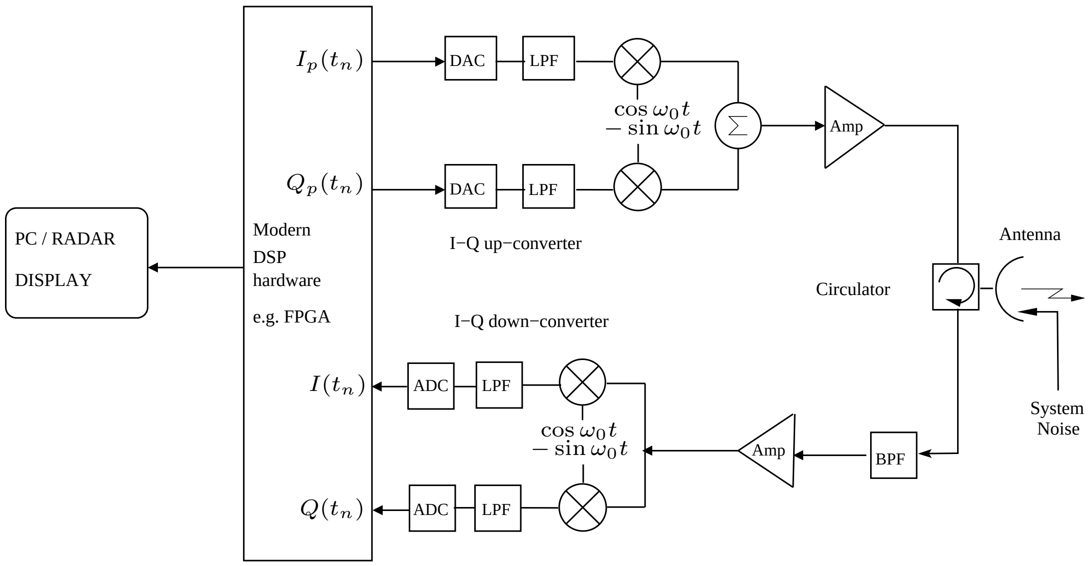
$$f_\theta(x) = \theta x,$$

   where θ is the parameter to be determined. Formulate this problem as a least-squares minimisation and use your result to find the optimum value of the parameter.

(5 marks)

2. **Radar/sonar signal processing**

The block diagram below shows the components of a radar. The radar transmits a chirp pulse of bandwidth 15 MHz, pulse length 100×10^{-6} seconds and centre frequency 1 GHz.



(a) Draw an equivalent analytic signal model of the radar. (2 marks)

(b) Explain how the signals generated by the DSP hardware (labelled I_p and Q_p) and those sampled by the DSP (labelled I and Q) relate to the transmitted RF waveform and the echo from an arbitrary scene (modelled by its impulse response). As part of your explanation, write down time and frequency domain expressions relating the signals, and include sketches of the frequency spectra of the signals to show their relationship. (4 marks)

(c) What is the minimum sample rate required for the DAC's and ADC's. (1 mark).

(d) What properties of the transmitted pulse determine (i) the resolution of the radar (ii) the SNR of the processed range data? (2 marks)

(e) What are the primary advantages of transmitting a CHIRP pulse compared to a monochrome pulse? (2 marks)

(f) A digital signal processing algorithm must be developed for pulse compression and display of the received echoes. The processor should optimize signal to noise ratio *and* keep the side-lobes low. Describe the digital signal processing steps that you would apply to produce a range profile for display from the echo from one transmitted pulse. The inputs to the processor are the sampled I and Q signals. Give details on the filters used, and how you would formulate them. Sketch the expected display output for the case of two point targets in the scene. (4 marks)

Fourier transform properties

Sequences $x[n]$, $y[n]$	Transforms $X(e^{j\omega})$, $Y(e^{j\omega})$	Property
$ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$	Linearity
$x[n - n_d]$	$e^{-j\omega n_d} X(e^{j\omega})$	Time shift
$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega-\omega_0)})$	Frequency shift
$x[-n]$	$X(e^{-j\omega})$	Time reversal
$nx[n]$	$j\frac{dX(e^{j\omega})}{d\omega}$	Frequency diff.
$x[n] * y[n]$	$X(e^{-j\omega})Y(e^{-j\omega})$	Convolution
$x[n]y[n]$	$\frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega-\theta)})d\theta$	Modulation

Common Fourier transform pairs

Sequence	Fourier transform
$\delta[n]$	1
$\delta[n - n_0]$	$e^{-j\omega n_0}$
$1 \quad (-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
$a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{1-ae^{-j\omega}}$
$u[n]$	$\frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
$(n+1)a^n u[n] \quad (\lvert a\rvert < 1)$	$\frac{1}{(1-ae^{-j\omega})^2}$
$\frac{\sin(\omega_c n)}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1 & \lvert\omega\rvert < \omega_c \\ 0 & \omega_c < \lvert\omega\rvert \le \pi \end{cases}$
$x[n] = \begin{cases} 1 & 0 \le n \le M \\ 0 & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
$e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$

Common z-transform pairs

Sequence	Transform	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$-u[-n-1]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert < 1$
$\delta[n - m]$	z^{-m}	All z except 0 or ∞
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$na^n u[n]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert > \lvert a\rvert$
$-na^n u[-n-1]$	$\frac{az^{-1}}{(1-az^{-1})^2}$	$\lvert z\rvert < \lvert a\rvert$
$\begin{cases} a^n & 0 \le n \le N-1, \\ 0 & \text{otherwise} \end{cases}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$\lvert z\rvert > 0$
$\cos(\omega_0 n)u[n]$	$\frac{1-\cos(\omega_0)z^{-1}}{1-2\cos(\omega_0)z^{-1}+z^{-2}}$	$\lvert z\rvert > 1$
$r^n \cos(\omega_0 n)u[n]$	$\frac{1-r\cos(\omega_0)z^{-1}}{1-2r\cos(\omega_0)z^{-1}+r^2 z^{-2}}$	$\lvert z\rvert > r$