EEE401F EXAM
DIGITAL SIGNAL PROCESSING

University of Cape Town
Department of Electrical Engineering

June 2004

3 hours

Information

- The exam is closed-book.
- There are two parts to this exam.
- **Part A** has *six* questions totalling 70 marks. You must answer all of them.
- **Part B** has *two* questions totalling 30 marks. You must answer all of them.
- You have 3 hours.

PART A

Answer all of the following questions.

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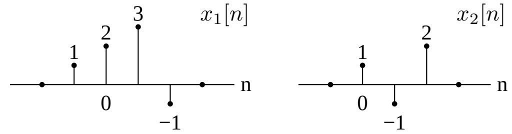
1. You are given the two signals $x_1[n]$ and $x_2[n]$ shown below. The signal values are zero outside of the range of n shown:



   (a) Use graphical methods to find the linear convolution $y[n] = x_1[n] * x_2[n]$.

   (b) Use the expression for the convolution sum directly to find the values of $y[n]$ for $n = -1$ and $n = 1$.

(10 marks)

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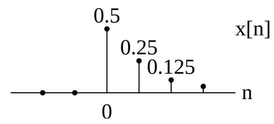
2. Consider the aperiodic exponentially decaying signal shown below:



   (a) Find the Fourier transform of the $x[n]$ and sketch its magnitude over the range -2π to 2π. Does the signal have predominantly low or high-frequency content?

   (b) Calculate the 4-point DFT $X[k]$ of the signal.

   (c) If $\tilde{x}[n]$ is the inverse DFT of $X[k]$, what is the value of $\tilde{x}[-2]$ and $\tilde{x}[4]$?

(10 marks)

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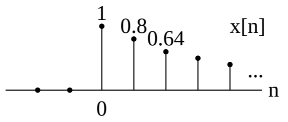
3. (a) Find the z-transform of the exponentially-decaying signal



(b) What is the z-transform of the signal $y[n] = x[n] * x[n]$? Use this result, along with the transform pair

$$na^n u[n] \stackrel{z}{\longleftrightarrow} \frac{az^{-1}}{(1 - az^{-1})^2},$$

to find $y[n]$.

(c) Use either a power-series expansion or the method of long division to find the first 4 values of the causal signal corresponding to the z-transform

$$X(z) = \frac{1}{1.2 + z}.$$

Where is the region of convergence in the z-plane?

(15 marks)

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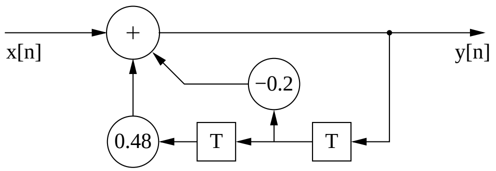
4. The figure below shows a second-order system:



(a) Write down an expression for the z-transform of the system.

(b) Under the assumption that the system is causal, draw a pole-zero plot of the system in the z-plane. Is the system stable?

(c) Find the impulse response of the system under the assumptions given. You may need the transform pair

$$a^n u[n] \stackrel{z}{\longleftrightarrow} \frac{1}{1 - az^{-1}}, \qquad |z| > |a|.$$

(15 marks)

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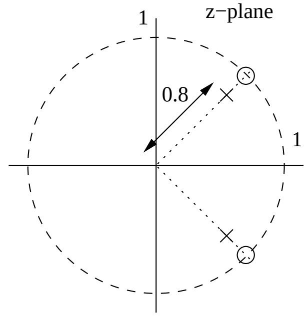
5. The pole-zero plot for a filter is as follows:



(a) Use graphical methods to roughly sketch the magnitude and phase response of the filter.

(b) Describe the effect the filter will have on an input signal.

(c) What are the possible regions of convergence for the filter? Which will correspond to a causal filter, and which to a stable filter?

(10 marks)

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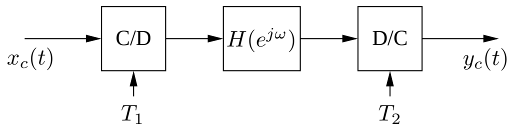
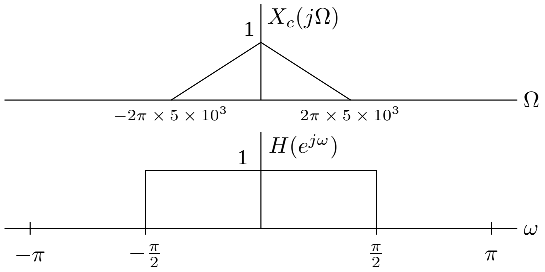
6. In the system



$X_c(j\Omega)$ and $H(e^{j\omega})$ are as shown below:



Sketch and label the Fourier transform of $y_c(t)$ for each of the following cases:

(a) $1/T_1 = 1/T_2 = 10^4$.

(b) $1/T_1 = 1/T_2 = 2 \times 10^4$.

(c) $1/T_1 = 2 \times 10^4, 1/T_2 = 10^4$.

(d) $1/T_1 = 10^4, 1/T_2 = 2 \times 10^4$.

(10 marks)

PART B

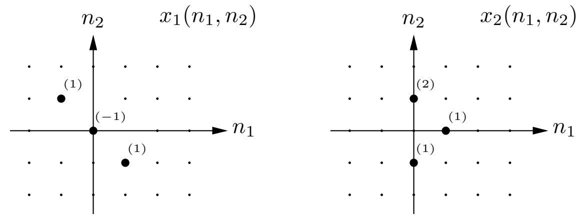
Answer *both* of the following two questions. Each question counts 15 marks.

1. **Multidimensional signal and image processing**

(a) Explain why a periodic sequence in 2-D is more complex than for 1-D.

(5 marks)

(b) Use the method of your choice to find the 2-D convolution between the following two signals:



(5 marks)

(c) Describe in detail how a charge-coupled device (CCD) creates a signal.

(5 marks)

2. **Speech processing**

(a) Radix-3 Decimation in time FFT algorithm:

i. Write an expression for $X[k]$ involving three $N/3$-DFTs.

ii. What is the advantage of a radix-3 FFT for a $N = 3^\nu$ length signal as compared to a direct computation of the DFT?

(5 marks)

(b) True or False:

i. A radix-2 Decimation-in-time FFT algorithm will be faster than a radix-2 Decimation-in-frequency FFT algorithm.

ii. A spectrogram contains 3-D information about a speech signal.

iii. Estimating nasals parameters is one of the weakness of LPC model.

iv. Zero padding a signal before computing its DFT will increase its frequency resolutions.

v. Vowels have distinct formants.

(5 marks)

(c) The following are three different signals $x_i[n]$ that are the sum of two sinusiods:

$$x_1[n] = \cos(\pi n/4) + \cos(17\pi n/64)$$
$$x_2[n] = \cos(\pi n/4) + 0.8\cos(21\pi n/64)$$
$$x_3[n] = \cos(\pi n/4) + 0.0001\cos(21\pi n/64)$$

We wish to estimate the spectrum of each of these signals using the 64-point DFT with a 64-point rectangular window $w[n]$. Indicate for which of the signals you would expect to see two distinct spectral peaks.

(5 marks)