EEE2035F: Signals and Systems I

Class Test 1

19 March 2012

SOLUTIONS

Name:

Student number:

Information

- The test is closed-book.
- This test has *four* questions, totalling 25 marks.
- Answer *all* the questions.
- You have 45 minutes.

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1. (10 marks) Suppose $x(t) = e^{-t}u(t)$. Sketch the following:

   (a) $y_1(t) = x(t)$

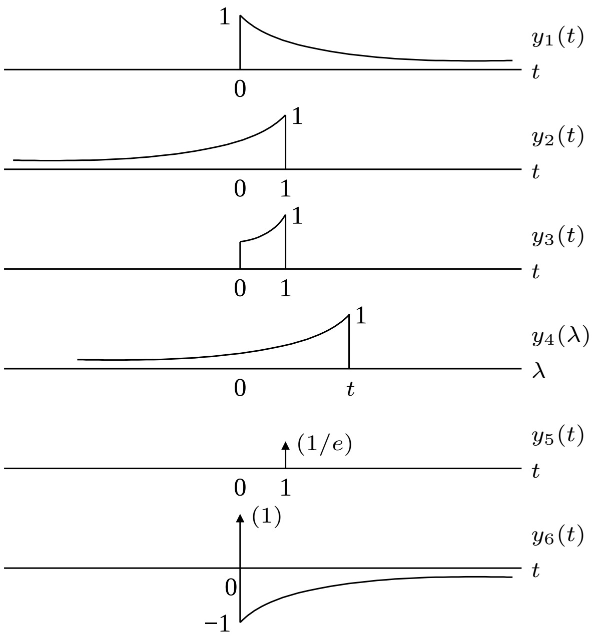
   (b) $y_2(t) = x(-t + 1)$

   (c) $y_3(t) = x(-t + 1)u(t)$

   (d) $y_4(\lambda) = x(t - \lambda)$

   (e) $y_5(t) = x(t)\delta(t - 1)$

   (f) $y_6(t) = \frac{d}{dt}x(t)$



2. (5 marks) Suppose the output $y(t)$ of a system is related to the input $x(t)$ via the relationship

$$y(t) = x(t) + 1.$$

(a) Is the system linear?

(b) Is the system time invariant?

(a) We can show by counterexample that the system is not homogeneous, and is therefore not linear. Consider the input $x_1(t) = u(t)$. The following is a valid input-output pair:

$$x_1(t) \longrightarrow y_1(t) = \{x_1(t)\} + 1 = u(t) + 1.$$

However, using the input $x_2(t) = 2u(t)$ we get the following valid input-output pair:

$$x_2(t) \longrightarrow y_2(t) = \{x_2(t)\} + 1 = 2u(t) + 1.$$

Clearly for the inputs we have $x_2(t) = 2x_1(t)$, by for the outputs we don't have $y_2(t) = 2y_1(t)$. Therefore the system is not homogeneous, and is not linear.

(b) Assuming that the input is $x_1(t)$, the output will be $y_1(t) = x_1(t) + 1$. The following input-output pair is therefore valid:

$$x_1(t) \longrightarrow y_1(t) = x_1(t) + 1.$$

Suppose now that we consider the shifted input $x(t) = x_1(t - \lambda)$ for some λ. The output will be

$$y(t) = x(t) + 1 = \{x_1(t - \lambda)\} + 1 = x_1(t - \lambda) + 1 = y_1(t - \lambda).$$

A shift in the input therefore always causes a corresponding shift in the output, so the system is time invariant.

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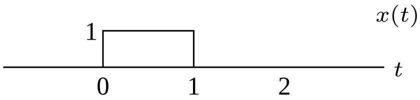
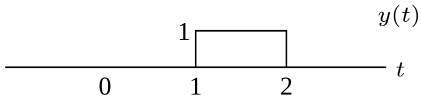
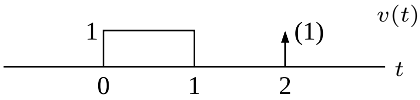
3. (5 marks) Suppose you're given the signals

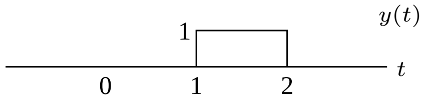




(a) Use the method of your choice to find $w(t) = x(t) * y(t)$.

(b) Use the result from the previous question to find $f(t) = y(t) * v(t)$ for these signals:

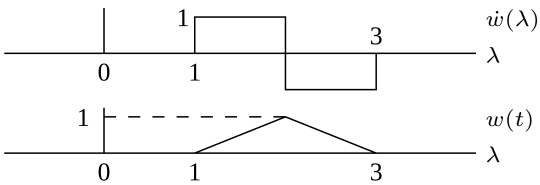




(a) The derivative property implies that $\dot{w}(t) = \dot{x}(t) * y(t)$. However, $\dot{x}(t) = \delta(t) - \delta(t-1)$, so using the properties of convolution with the delta function we get

$$\dot{w}(t) = [\delta(t) - \delta(t-1)] * y(t) = y(t) - y(t-1).$$

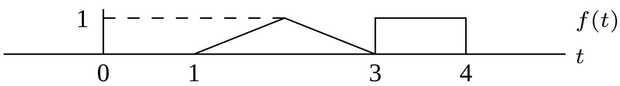
The signal $w(t)$ can then be found by indefinite integration: $w(t) = \int_{-\infty}^{t} \dot{w}(\lambda) d\lambda$. The signals of interest are shown below:



(b) Evidently $v(t) = x(t) + \delta(t-2)$, so the required signal is

$$\begin{aligned} f(t) &= y(t) * v(t) = y(t) * [x(t) + \delta(t-2)] \\ &= y(t) * x(t) + y(t-2) = w(t) + y(t-2), \end{aligned}$$

where $w(t)$ was found in the previous part. Thus $f(t)$ is given below:



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4. (5 marks) A unit step input is applied to a LTI system, and results in the following response:

$$y(t) = \frac{1}{2}tu(t) - \frac{1}{20}(1 - e^{-10t})u(t).$$

(a) Find and plot $\frac{d}{dt}y(t)$.

(b) Use the derivative property of convolution to find the impulse response of the system.

(a) We can rewrite $y(t)$ as follows:

$$y(t) = \begin{cases} \frac{1}{2}t - \frac{1}{20}(1 - e^{-10t}) & t \geq 0 \\ 0 & t < 0. \end{cases}$$

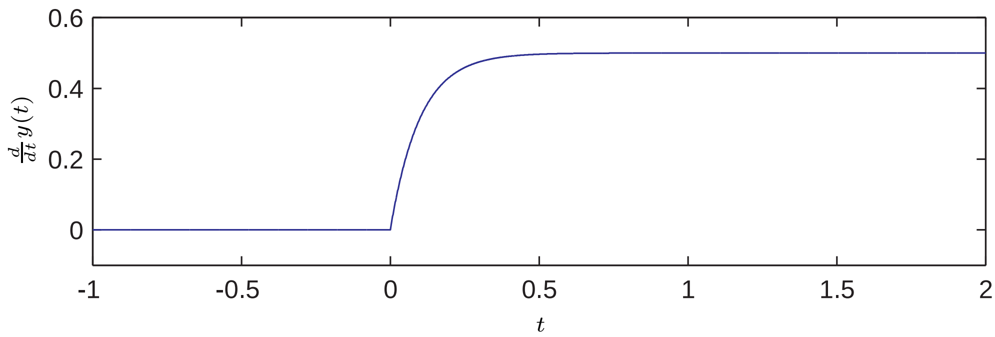
Note that there is no discontinuity at $t = 0$ since

$$\lim_{t \to 0} \left(\frac{1}{2}t - \frac{1}{20}(1 - e^{-10t}) \right) = 0.$$

For $t < 0$ we have $\frac{d}{dt}y(t) = 0$ and for $t \geq 0$ we have

$$\frac{d}{dt}y(t) = \frac{d}{dt}\left(\frac{1}{2}t - \frac{1}{20} + \frac{1}{20}e^{-10t} \right) = \frac{1}{2} + \frac{1}{20}(-10)e^{-10t} = \frac{1}{2}(1 - e^{-10t}),$$

which looks like



(b) If $u(t) \longrightarrow y(t)$ is a valid input-output pair for a LTI system, then from the derivative property we know that

$$\frac{d}{dt}u(t) \longrightarrow \frac{d}{dt}y(t)$$

is also valid. However, since $\frac{d}{dt}u(t) = \delta(t)$ we have $\delta(t) \longrightarrow \frac{d}{dt}y(t)$, so by definition the impulse response is

$$h(t) = \frac{d}{dt}y(t),$$

which was plotted in the previous part.