EEE235F Class Test

15 April 2005

Name:

Student number:

Information

- The test is closed-book.
- This test has *five* questions, totalling 50 marks.
- Answer *all* the questions.
- You have 45 minutes.

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1. (10 marks) Are the following signals periodic? If so, what is the fundamental period and frequency?

   (a) $x(t) = \cos(\frac{\pi}{3}t) + 3\sin(\frac{\pi}{4}t)$

   (b) $x(t) = e^{j(\frac{\pi}{2}t-1)}$.

   (a) Find smallest T such that $x(t) = x(t+T)$, or

$$\begin{aligned}\cos(\frac{\pi}{3}t) + 3\sin(\frac{\pi}{4}t) &= \cos(\frac{\pi}{3}(t+T) + 3\sin(\frac{\pi}{4}(t+T)) \\ &= \cos(\frac{\pi}{3}t + \frac{\pi}{3}T) + 3\sin(\frac{\pi}{4}t + \frac{\pi}{4}T)\end{aligned}$$

   The condition above is satisfied for $\frac{\pi}{3}T = 2k\pi$ and $\frac{\pi}{4}T = 2m\pi$, for k and m integers. Thus if we can find integers k and m such that $T = 6k$ and $T = 8m$, then the signal is periodic. The smallest integers satisfying $T = 6k = 8m$ are $k = 4$ and $m = 3$, so the signal is periodic with fundamental period $T = 6(4) = 8(3) = 24$. The fundamental frequency is therefore $f = 1/T = 1/24$ Hz.

   (b) Find smallest T such that

$$e^{j(\frac{\pi}{2}t-1)} = e^{j(\frac{\pi}{2}(t+T)-1)} = e^{j(\frac{\pi}{2}t+\frac{\pi}{2}T-1)} = e^{j(\frac{\pi}{2}t-1)}e^{j\frac{\pi}{2}T}.$$

   The smallest value of T such that this is true is $T = 4$.

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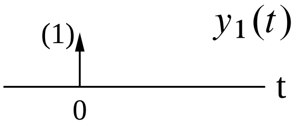
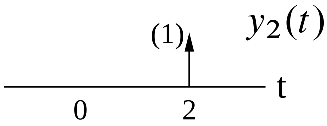
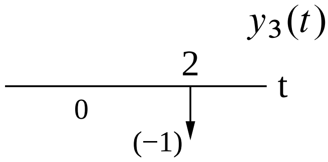
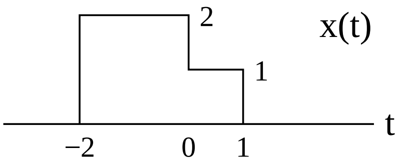
2. (10 marks) Suppose $y_1(t)$, $y_2(t)$ and $y_3(t)$ are as shown below:

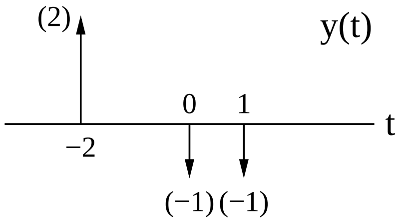






If $x(t)$ and $y(t)$ are





then sketch

(a) $x(t) * y_1(t)$

(b) $x(t) * y_2(t)$

(c) $x(t) * y_3(t)$

(d) $y(t) * y_1(t)$

(e) $y(t) * y_2(t)$

(f) $y(t) * y_3(t)$.

The solutions are as follows — see next page for sketches.

(a) $x(t) * y_1(t) = x(t) * \delta(t) = x(t)$, since $\delta(t)$ is the identity element of convolution.

(b) $x(t) * y_2(t) = x(t) * \delta(t-2) = x(t-2)$, since shifting one signal by 2 shifts the resulting convolution by 2.

(c) $x(t) * y_3(t) = x(t) * -1\delta(t-2) = -1(x(t) * \delta(t-2)) = -x(t-2)$, since the convolution operator is linear.

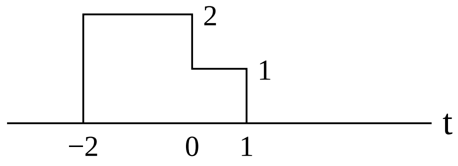
(d) $y(t) * y_1(t) = y(t) * \delta(t) = y(t)$.

(e) $y(t) * y_2(t) = y(t) * \delta(t-2) = y(t-2)$.

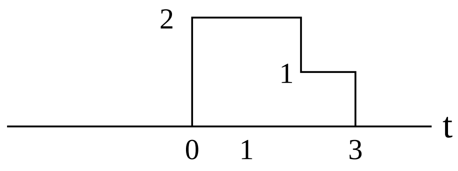
(f) $y(t) * y_3(t) = y(t) * -1\delta(t-2) = -1(y(t) * \delta(t-2)) = -y(t-2)$.

Sketches are as follows:

- Questions (a), (b), and (c)

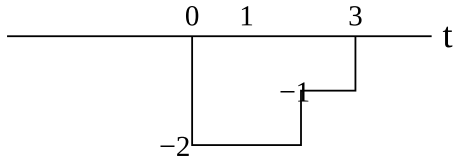


$x(t) * \delta(t)$

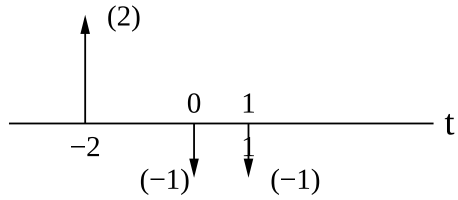


$x(t) * \delta(t-2)$

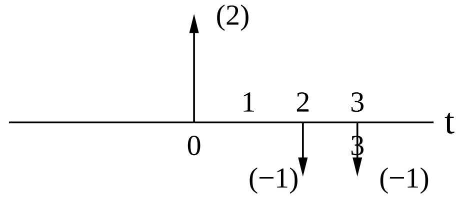
$x(t) * -\delta(t-2)$



- Questions (d), (e), and (f)

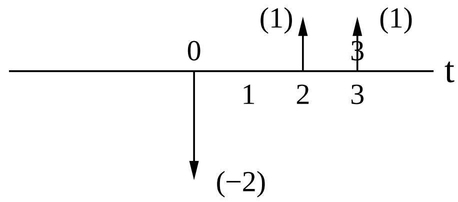


$y(t) * \delta(t)$

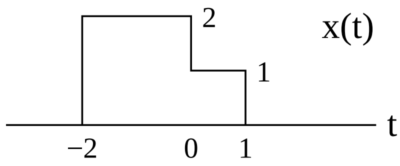


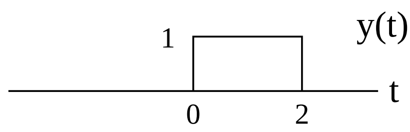
$y(t) * \delta(t-2)$

$y(t) * -\delta(t-2)$



3. (10 marks) Use the derivative property of convolution to find $w(t) = x(t) * y(t)$, where





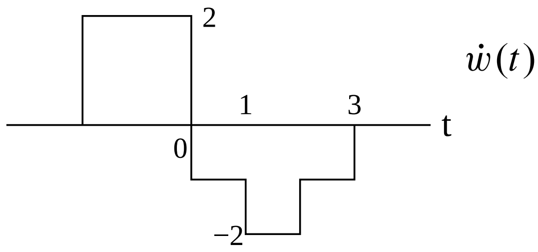
If $w(t) = x(t) * y(t)$, then the derivative property of convolution states that $\dot{w}(t) = x(t) * \dot{y}(t)$. However, $\dot{y}(t) = \delta(t) - \delta(t-2)$ (generalised derivative of $y(t)$), so

$$\dot{w}(t) = x(t) * \dot{y}(t) = x(t) * (\delta(t) - \delta(t-2)) = [x(t) * \delta(t)] + [x(t) * -\delta(t-2))]$$

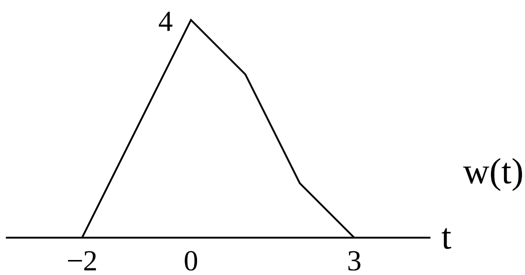
From the results of the previous section (or otherwise) we know that this equals

$$\dot{w}(t) = x(t) - x(t-2),$$

so $\dot{w}(t)$ looks like



Integrating once gives the required answer:



4. (10 marks) Consider a continuous-time LTI system described by

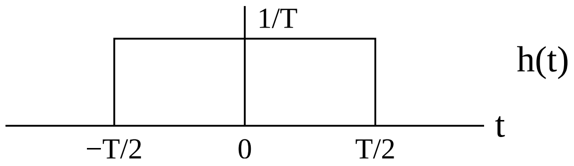
$$y(t) = T\{x(t)\} = \frac{1}{T}\int_{t-T/2}^{t+T/2} x(\tau)d\tau.$$

(a) Find and sketch the impulse response $h(t)$ of the system.

(b) Is the system causal?

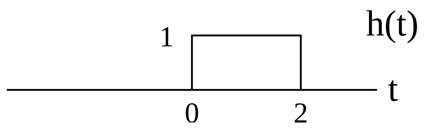
(a) Impulse response is the output of the system when the input is the impulse: $x(t) = \delta(t)$. Thus the impulse response $h(t)$ is

$$h(t) = T\{\delta(t)\} = \frac{1}{T}\int_{t-T/2}^{t+T/2} \delta(\tau)d\tau = \begin{cases} \frac{1}{T} & -\frac{T}{2} < t < \frac{T}{2} \\ 0 & \text{otherwise.} \end{cases}$$



(b) The system is not causal, since the impulse response is nonzero for some values of t.

5. (10 marks) Suppose a LTI system has impulse response



(a) What is the response of the system to the complex signal

$$x_1(t) = e^{j\omega t}$$

for some fixed ω?

(b) Hence, by writing $\cos(x)$ in terms of complex exponentials, find the response of the system to

$$x_2(t) = \cos(\omega t).$$

Note that in this case the result should be *real valued*, so some simplification may be necessary.

(a) Direct convolution gives the result:

$$y_2(t) = \int_{-\infty}^{\infty} h(\tau)x_1(t-\tau)d\tau = \int_0^2 e^{j\omega(t-\tau)}d\tau = e^{j\omega t}\int_0^2 e^{-j\omega\tau}d\tau$$
$$= e^{j\omega t}\left[\frac{1}{-j\omega}e^{-j\omega\tau}\right]_{\tau=0}^{\tau=2} = -\frac{1}{j\omega}e^{j\omega t}\left(e^{-j\omega 2}-1\right) = \frac{1-e^{-j\omega 2}}{j\omega}.$$

(b) The signal $x_2(t) = \cos(\omega t)$ can be written as

$$x_2(t) = \frac{1}{2}\left(e^{j\omega t}+e^{-j\omega t}\right) = \frac{1}{2}e^{j\omega t} + \frac{1}{2}e^{-j\omega t}.$$

Because the system is linear, the response to $x_2(t)$ will be the sum of the responses to each of the terms above. Now from part (a) the response of the system to input $e^{j\omega t}$ is $e^{j\omega t}(1-e^{-j\omega 2})/(j\omega)$ so the response to $1/2e^{j\omega t}$ will be $e^{j\omega t}(1-e^{-j\omega 2})/(j2\omega)$. Similarly, the response to

$1/2e^{j(-\omega)t}$ will be $e^{j\omega t}(1-e^{-j(-\omega)2})/(j2(-\omega))$. Thus the output will be

$$
\begin{aligned}
y(t) &= e^{j\omega t}(1-e^{-j\omega 2})/(j2\omega) + e^{j(-\omega)t}(1-e^{-j(-\omega)2})/(j2(-\omega)) \\
&= \frac{1}{j2\omega}\left(e^{j\omega t} - e^{j\omega(t-2)} - e^{-j\omega t} + e^{-j\omega(t-2)}\right) \\
&= \frac{-1}{j2\omega}\left(\left[e^{j\omega(t-2)} - e^{-j\omega(t-2)}\right] - \left[e^{j\omega t} - e^{-j\omega t}\right]\right) \\
&= -\frac{1}{\omega}\left(\sin(\omega(t-2)) - \sin(\omega t)\right).
\end{aligned}
$$
